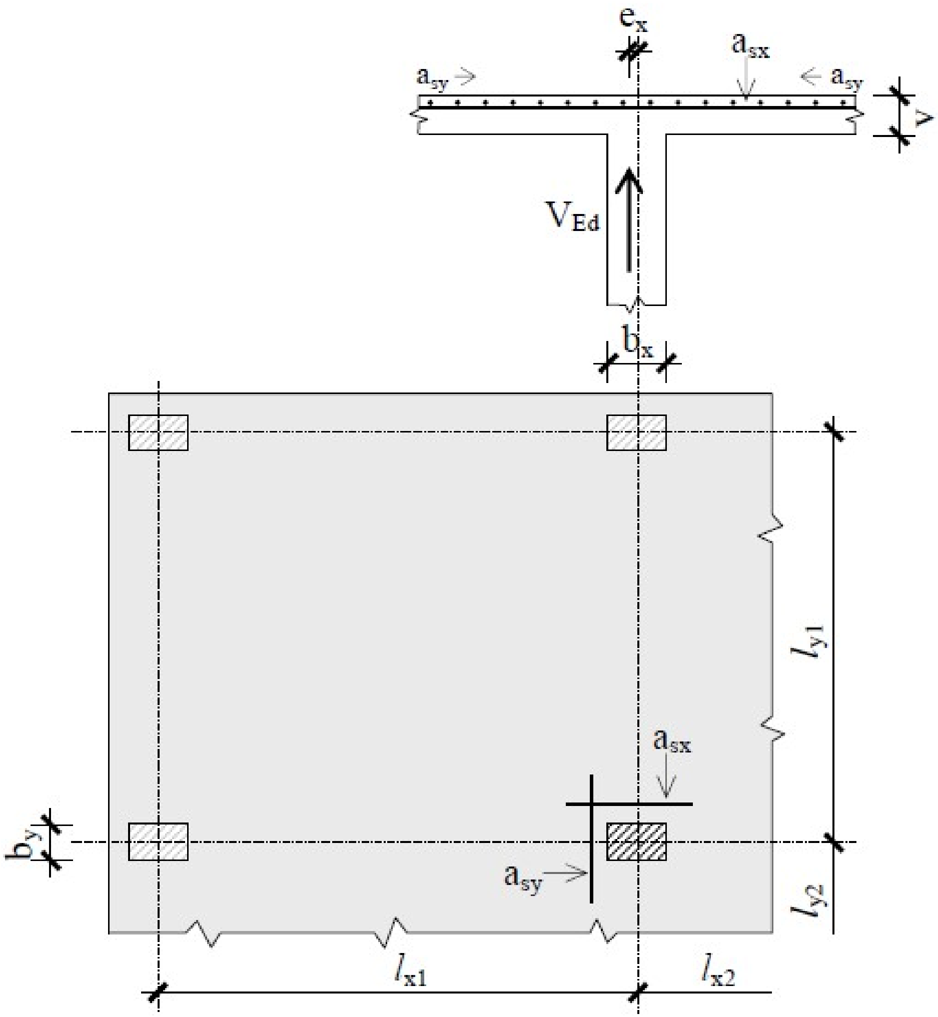
5. punchig shear in flat slab

a.) Check the flat slab bellow for punching shear at an internal column!
Horizontal forces are taken by appropriate shear walls .

b.) Design the punching reinforcement using ∅ 12 double-headed studs as shear reinforcement

Consider the detailing rules!



Data:

$V_{Ed} = 467$ kN

$e_x = 95$ mm $\qquad e_y = 105$ mm

$b_x = 400$ mm

$b_y = 200$ mm

$v = 200$ mm

$l_{x1} = 7{,}00$ m $\qquad l_{x2} = 7{,}60$ m

$l_{y1} = 6{,}00$ m $\qquad l_{y2} = 6{,}80$ m

Concrete: C25/30

Steel: S500

Tensile (top) reinforcement:

$a_{sx} = 2513$ mm²/m $\qquad d_x = 131$ mm

$a_{sy} = 2681$ mm²/m $\qquad d_y = 147$ mm

Solution

a.) Check the floor slab at an internal column!
The horizontal loads are resisted by a shear wall system.

I. Data

Characteristic strength of the connrete : $f_{ck} = 25 \frac{N}{mm^2}$

Design strength of the connrete: $f_{cd} = \frac{f_{ck}}{1.5} = 16.7 \cdot \frac{N}{mm^2}$

Characteristic strength of the steel: $f_{yk} = 500 \frac{N}{mm^2}$

Design strength of the steel: $f_{yd} = \frac{f_{yk}}{1.15} = 434.8 \cdot \frac{N}{mm^2}$

Distance of the columns: $l_{x1} = 7.00m$ $l_{x2} = 7.60m$

$l_{y1} = 6.00m$ $l_{y2} = 6.80m$

Column section: $b_x = 400mm$ $b_y = 200mm$

Slab thickness: $v = 200mm$

Top reinforcement of the slab: $a_{sx} = 2513mm^2$ $a_{sy} = 2681mm^2$

Effective height: $d_x = 131mm$ $d_y = 147mm$

Design value of the shar force: $V_{Ed} = 467kN$

II. Basic calculations

Effective height: $$d = \frac{d_x + d_y}{2} = 139 \cdot mm$$

Rinfoercment ratio of the slab: $$\rho_{lx} = \frac{a_{sx}}{1000mm \cdot d_x} = 0.0192$$

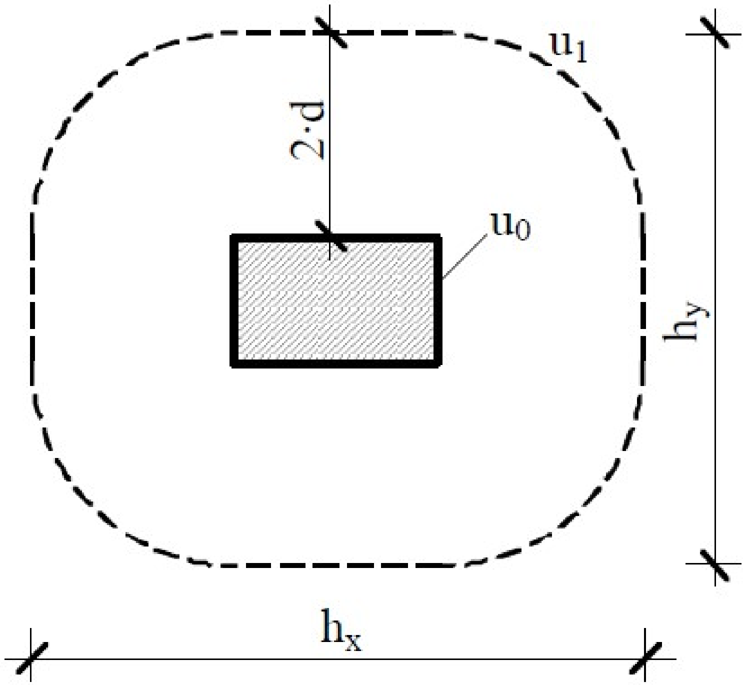
$$\rho_{ly} = \frac{a_{sy}}{1000mm \cdot d_y} = 0.0182$$

$$\rho_l = \sqrt{\rho_{lx} \cdot \rho_{ly}} = 0.0187 \le 0,02$$

III. Design value of the specific shear force

Columns' perimeter: $u_0 = 2 \cdot b_x + 2 \cdot b_y = 1.2\,m$

the first punching perimeter: $u_1 = 2 \cdot b_x + 2 \cdot b_y + 2 \cdot 2 \cdot d \cdot \pi = 2.95\,m$



Eccentricity of the shear force: $e_x = 95mm$ $e_y = 105mm$

Calculation of β that takes the eccentricity of the shear force into assumption:

There are several methods for this in the EC2:

A.) If the spans of the neighboiring slab panels do not differ mor than 25% and the horizontal loads are supported by an adeuate stiffening system than the following approximate values could be used for consideration the eccentricity:

- internal column: β=1,15
- side column: β=1,4
- corner column: β=1,5.

Now an internal column is analysed. Difference of spans:

$$\frac{l_{x2}}{l_{x1}} - 1 = 8.6 \cdot \% \qquad \text{and} \qquad \frac{l_{y2}}{l_{y1}} - 1 = 13.3 \cdot \%$$

The difference is smaller then 25%. The horizontal forces are taken by shear walls. This way the eccentricity factor:

$$\beta_A = 1.15$$

B.) In case of biaxial eccentricity there is a more accurate - however still approximate - method available:

$$h_x = b_x + \cdot 2 \quad d \quad = 678 \cdot mm \text{ (see the previous figure)}$$

$$h_y = b_y + 2 \cdot d = 478 \cdot mm$$

$$\beta_B = 1 + 1.8 \cdot \sqrt{\left(\frac{e_x}{h_x}\right)^2 + \left(\frac{e_y}{h_y}\right)^2} = 1.47$$

In the further calculations for β will be used the value obtained from a more sophisticated analysis: β = 1.38

Design values of the pecific shear firce modified with the eccentricity:

Specific shear firce along the column's perimeter: $v_{Ed.0} = \beta \cdot \frac{V_{Ed}}{d \cdot u_0} = 3.87 \cdot \frac{N}{mm^2}$

Specific shear firce along the u_1 perimeter: $v_{Ed} = \beta \cdot \frac{V_{Ed}}{d \cdot u_1} = 1.58 \cdot \frac{N}{mm^2}$

IV. Shear capacity of the concrete

$$k = \min\left(1 + \sqrt{\frac{200}{d}}, 2\right) = 2$$ d is substituted in mm

Min. value of the specific shear resistance:

$$v_{min} = 0.035 \cdot k^{\frac{3}{2}} \cdot f_{ck}^{\frac{1}{2}} = 0.49 \frac{N}{mm^2}$$

The shear resistance of the concrete:

$$v_{Rd.c} = \frac{0.18}{1.5} \cdot k \cdot \left(100 \cdot \rho_l \cdot f_{ck}\right)^{\frac{1}{3}} = 0.86 \frac{N}{mm^2} \geq v_{min} = 0.49 \cdot \frac{N}{mm^2}$$

Checking:

$$v_{Rd.c} = 0.86 \cdot \frac{N}{mm^2} < v_{Ed} = 1.58 \cdot \frac{N}{mm^2}$$

Reinforcement is needed for punching shear!

V. Upper boundary of the shear resistance

Effectiveness factor:

$$\nu = 0.6 \cdot \left(1 - \frac{f_{ck}}{250}\right) = 0.54$$

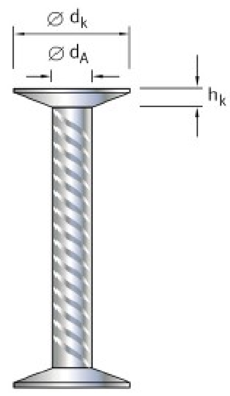
f_{ck} is in N/mm^2

Upper limit for punching resistance:

$$v_{Rd.max} = 0.5 \cdot \nu \cdot f_{cd} = 4.5 \cdot \frac{N}{mm^2} > v_{Ed.0} = 3.87 \cdot \frac{N}{mm^2}$$

The upper limit of the punching resistance of the slab (capacity of the compression strut) is adequate. Othewise the slab hight should be increased.

b.) Design the punching reinforcement by using ∅ 12 bars. Consider the construction rules!



a stud

VI. Calculation of the required punching reinforcement

Applied diameter: $\phi_W = 12\text{mm}$

The aera of 1 piece of stud: $A_{SW} = \phi_W^2 \cdot \frac{\pi}{4} = 113 \cdot \text{mm}^2$

angle of shear stud to the horizontal: $\alpha = 90°$

The effective strength of the studs: $f_{yd.ef} = 250 + 0.25 \cdot d = 285 \frac{\text{N}}{\text{mm}^2} \leq f_{yd}$

Punching resistance of the slab supplied with shear reinforcement:

$$v_{Rd.cs} = 0.75 \cdot v_{Rd.c} + 1.5 \cdot \frac{d}{s_r} \cdot \frac{n \cdot A_{sw} \cdot f_{yd.ef}}{d \cdot u_1} \cdot \sin(\alpha)$$

Where *n* is the numer of studs along a punching perimeter, s_r distance between the studs in radial direction.

Determine the radial distance of the studs according to constructional rules:

Allowable maximum radial distance: $s_{r.max} = 0.75 \cdot d = 104.3 \cdot mm$

The appliesd distance: $s_r = 100mm$

According to this the number of studs along a circle could be calculated:

It is required: $v_{Ed} \le v_{Rd.cs}$

Supposing equality:

$$n = \frac{v_{Ed} - 0.75 \cdot v_{Rd.c}}{1.5 \cdot d \cdot A_{sw} \cdot f_{yd.ef} \cdot \sin(\alpha)} \cdot s_r \cdot d \cdot u_1 = 5.67 \ db$$

Apply along a circle: $n_{app} = 6$ pieces

The punching reinforcement concicts of duble headed studs arranged in concentric circles. In a circle 6 pieces of studs are used. The radial distance of the circles is 100 mm.

Checking:

$$v_{Rd.cs} = 0.75 \cdot v_{Rd.c} + 1.5 \cdot \frac{d}{s_r} \cdot \frac{n_{alk} \cdot A_{sw} \cdot f_{yd.ef}}{d \cdot u_1} \cdot \sin(\alpha) = 1.63 \cdot \frac{N}{mm^2} > v_{Ed} = 1.58 \cdot \frac{N}{mm^2}$$

The quantity of the pubching reinforcement is adequate!

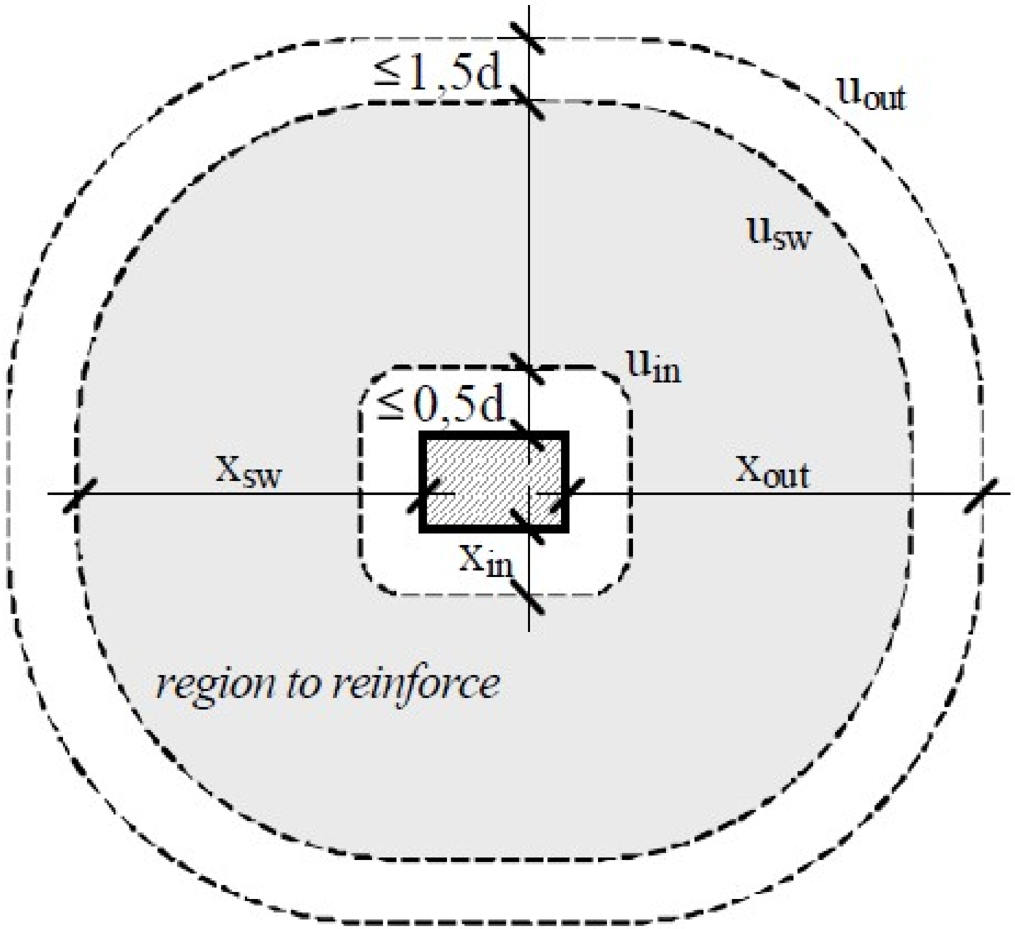
VII. Detailing rules

The checked perimeters:

u_{out} - where no reinforcement is needed. The concrete is capable to resist the shear. ($v_{Rd.c} = v_{Ed}$),

u_{sw} - the most extreme perimeter where punching reinforcement is still needed,

u_{in} - the first perimeter where punching reinforcement is needed.



- Distance of the most extreme $\mathbf{u_{out}}$ punching perimeter from the column

  Length of the punching perimeter being x distance from the column:

  $$u_i(x) = 2 \cdot b_x + 2 \cdot b_y + 2 \cdot x \cdot \pi$$

The specific shear force along the perimeter (*x* distance from the column):

$$v_{Ed.i}(x) = \beta \cdot \frac{V_{Ed}}{\cdot d \;\; u_i x(\;)}$$

At the out most perimeter the specific shear force is just equal with the shear resistence of the concrete:

$v_{Ed.i}(x) = v_{Rd.c}$ substituting: $$\beta \cdot \frac{V_{Ed}}{d \cdot \left(2 \cdot b_x + 2 \cdot b_y + 2 \cdot x \cdot \pi\right)} = v_{Rd.c}$$

Rearranged the distance of the u_{out} perimeter from the column:

$$x_{out} = \frac{\beta \cdot \frac{V_{Ed}}{d \cdot v_{Rd.c}} - 2 \cdot b_x - 2 \cdot b_y}{2 \cdot \pi} = 665 \cdot mm$$

- The distance of the $\mathbf{u}_{sw}$ perimeter from the column (still shear reinf. needed)

According to the deteiling rules the out most studs' distance from the u_{out} primeter must not be greater then 1.5*d :

$x_{sw} = x_{out} - 1.5 \cdot d = 456 \cdot mm$

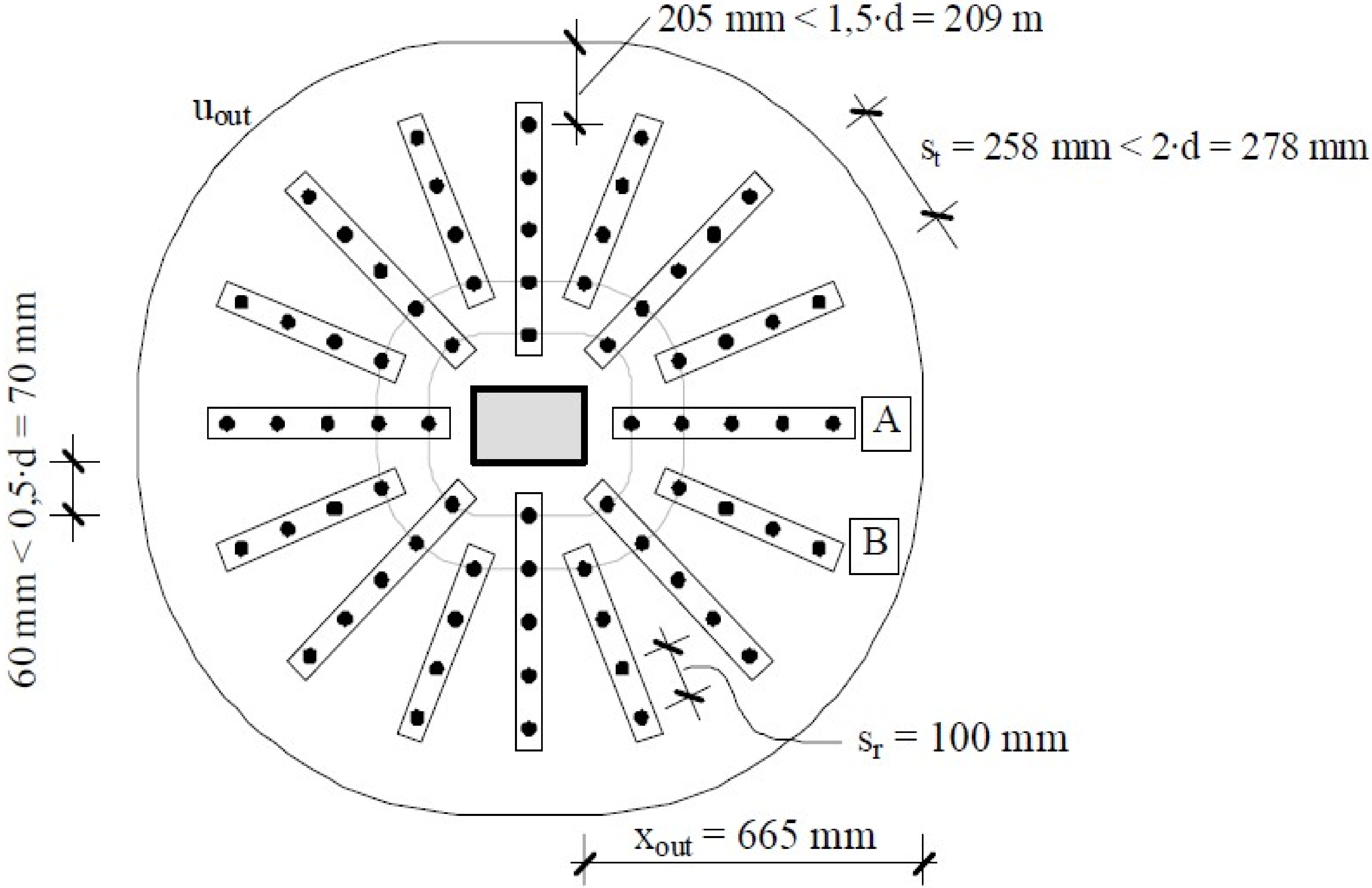
- The distance of the $\mathbf{u_{in}}$ perimeter from the column

According to the deteiling rules the studs (orother type of shear reinforcement) in the first perimeter could be placed in adistance max. 0,5*d* from the column, but closer than 0,3*d* to the column studs are not needed. For the first perimeter:

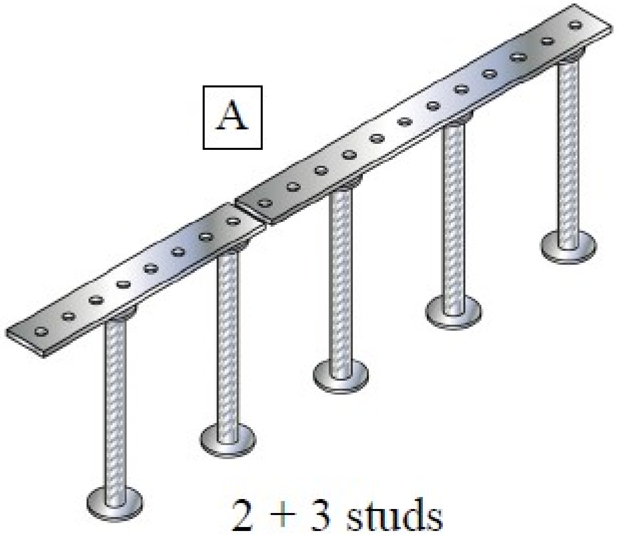
$$0.3 \cdot d = 41.7 \cdot mm \leq \quad x_{in} \leq \quad 0.5d = 69.5 \cdot mm$$

Let the radius of the first perimeter to be: $x_{in} = 60mm$.

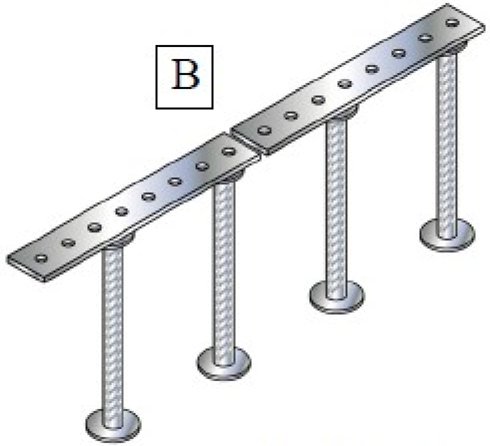
The applied punching reinforcement with the detailing rules:



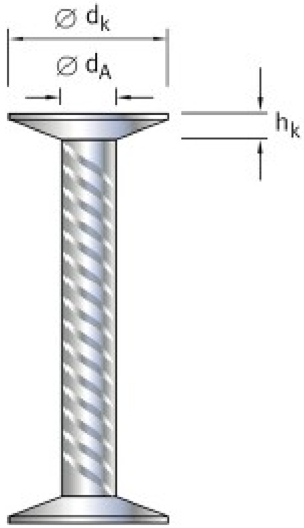
Prefabricated shear reinforcement consists of 2 or 3 studs welded to a steel rod. These could be combined:



2 + 3 studs



2 + 2 studs



a stud