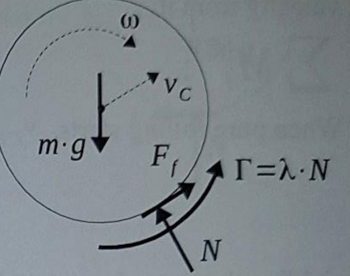
Example 3

A cylinder of radius $r=0.25$ m and mass $m=8$ kg rolls up against the slope of inclination $\alpha=30°$ without slipping. Initial velocity of the centre of the cylinder is $v_{C1}=14$ m/s. Find the distance covered by the cylinder if the factor of rolling resistance is $\lambda=0.04$ m.

Solution



The figure to the right shows all forces exerted upon the cylinder in a general instant of motion. Pure rolling implies that $v_C=\omega\cdot r$, and the moment Γ of rolling resistance is counterclockwise.

From a resolution perpendicular to the slope:

$$N-8\cdot 9.81\cdot\cos 30°=0 \rightarrow \quad N=67.97\text{ N},$$

which gives the moment of rolling resistance:

$$\Gamma=67.97\cdot 0.04=2.719\text{ Nm}$$

Kinetic energy is zero when the cylinder stops ($T_2=0$). In the initial configuration, kinetic energy can be obtained in two different ways:

$$T_1=\frac{1}{2}8\cdot 14^2+\frac{1}{2}\frac{8\cdot 0.25^2}{2}\left(\frac{14}{0.25}\right)^2=1176\text{ J} \quad \text{(translation plus rotation about the centre of mass)}$$

$$T_1=\frac{1}{2}\frac{3}{2}8\cdot 0.25^2\left(\frac{14}{0.25}\right)^2=1176\text{ J} \quad \text{(rotation about the instantaneous centre)}$$

The work of forces exerted on the body as well as of the moment of rolling resistance should be obtained with the following considerations. Forces perpendicular to the slope perform zero work; the bottom point has no velocity, so friction performs no work either. Pure rolling ensures that a distance *s* along the slope means an angle *s*/*R* (rad) of rotation of the cylinder: this is the angle the moment of rolling resistance performs a (negative) work on. The work-energy theorem states that

$$0-1176=-8\cdot 9.81\cdot\sin 30°\cdot s-2.719\cdot\frac{s}{0.25},$$

from which the distance covered until stop is $\boxed{s=23.47\text{ m}}$

Exercise 3 and mass of 10 kg (m = 10 kg)

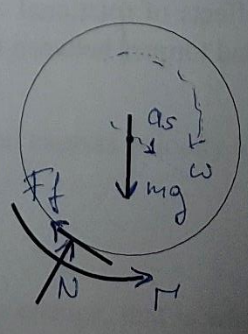
A cylinder of radius $r=0.25$ m is left on its own in rest on the top of a slope with an inclination of $\alpha=30°$. The cylinder starts moving downwards with a pure rolling motion, its factor of rolling resistance $\lambda=0.04$ m.

Find the distance needed for the centre of the cylinder to reach the velocity $v_C=10$ m/s.

$s=?$

Solution

Draw all forces exerted upon the body, as well as kinematic variables of the centre of mass and expected senses of accelerations into the figure.



Write Euler's first law in the direction perpendicular to the slope:

$\sum F_{i\perp}$: $m\cdot g\cdot\cos\alpha-N=0$

From this we get $10\cdot 9{,}81\cdot\cos 30°=N$

$N=$ 84,96 N $\qquad \Gamma=$ μN = 3,398 Nm

Calculate moments of inertia for both the point of contact and instantaneous centre of rotation:

$I_C = \frac{10 \cdot 0{,}25^2}{2} = 0{,}3125$ kgm² $\qquad I_0 = 0{,}9375$ kgm²

Write the kinetic enerqy both at the beginning and end of the motion:

$T_1 = 0$ $\qquad \omega_2 = \frac{v_S}{r} = \frac{10}{0{,}25} = 40$ rad/s

$T_2 = \frac{1}{2} I_0 \omega_2^2 = \frac{1}{2} \cdot 0{,}9375 \cdot \omega_2^2 = 750$ J

Distances (angles) that individual forces (moments) do work on:

$m \cdot g \cdot \cos\alpha$: 0 $\qquad m \cdot g \cdot \sin\alpha$: s

N: 0 $\qquad \Gamma$: $\frac{s}{r} = \frac{s}{0{,}25}$ (φ rotation)

F_f: 0

The work-energy theorem:

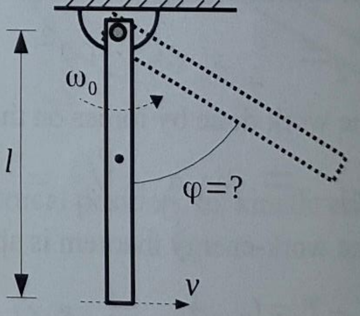
$T_2 - T_1 = L_{1-2}$: $\quad 750 - 0 = 10 \cdot 9{,}81 \cdot \sin 30^\circ \cdot s - \underbrace{3{,}398}_{\Gamma} \cdot \frac{s}{0{,}25}$

The distance that was looked for is then

$s=$ 21,15 m

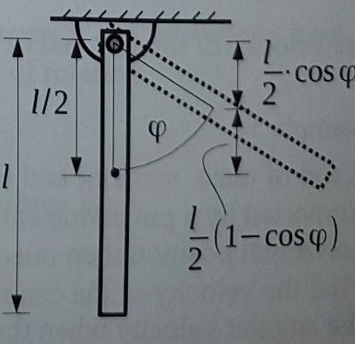
Example 4

A rod of mass $m = 5\,\text{kg}$ and length $l = 1.6\,\text{m}$ swings about a pinned support. While passing the bottom vertical configuration, the velocity of its lowest point is $v = 3\,\text{m/s}$. Find the maximum angle φ of deviation.



Solution

Angular acceleration could be found from forces exerted on the rod for any particular angle of deviation but κ would not be constant and the solution of the resultant differential equation would not be easy at the present level of discussion, that is why the work-energy theorem is used here.



The kinetic energy at the instant of stop is $T_2 = 0$ J, while for the bottom vertical configuration it can be directly obtained from the rotation about the pin. At the same time, angular velocity is calculated from the velocity of the bottom point:

$$\omega_0 = v/l = 3/1.6 = 1.875\,\text{rad/s}$$

Kinetic energy is therefore $T_1 = \frac{1}{2} \frac{5 \cdot 1.6^2}{3} 1.875^2 = 7.5\,\text{J}$

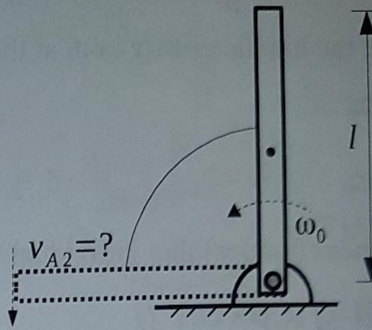
Forces transmitted at the pin do zero work because the pin is fixed. The work of gravity depends on the vertical translation of the centre of mass of the rod and can be found based on the second figure: the translation happens upwards, making the work of a force directed downwards to be negative. The theorem now reads:

$$0-7.5=-5\cdot 9.81\cdot\frac{1.6}{2}(1-\cos\varphi),$$

which yields the maximum angular deviation as $\boxed{\varphi=36.01°}$

Exercise 4

A rod of mass $m=8$ kg and length $l=1.8$ m is supported by a pin at its bottom. The rod passes its upright position with an angular velocity $\omega_0=4$ rad/s.
Find the velocity of the moving endpoint when the rod reaches a horizontal position.



Solution

If the angular velocity of the body in its horizontal position is known, then the velocity at any other point of the body can be obtained.

The body rotates about a fixed axis, so its kinetic energy can be calculated from the formula related to the instantaneous centre of rotation; the corresponding moment of inertia is

$$I_0=\frac{ml^2}{3}=\frac{8\cdot 1{,}8^2}{3}=8{,}64\ \text{kgm}^2$$

Thus, kinetic energy in at the initial instant:

$$T_1=\frac{1}{2}I_0\,\omega_0^2=\frac{1}{2}\cdot 8{,}64\cdot 4^2=69{,}12\ \text{J}$$

The kinetic energy just before the collision, assuming ω to be the current angular velocity:

$$T_2=\frac{1}{2}8{,}64\cdot\omega^2$$

The work done by forces on the body:

$$L_{1-2}=mg\cdot l/2=8\cdot 9{,}81\cdot 0{,}9=70{,}63\ \text{J}$$

The work-energy theorem is applied as

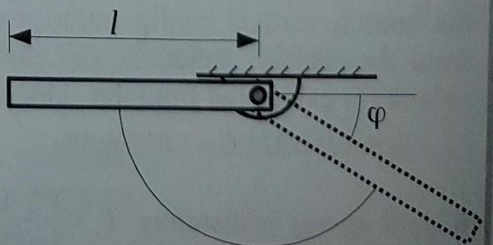
$$T_2-T_1=L_{1-2}\rightarrow\quad \frac{1}{2}\cdot 8{,}64\cdot\omega^2-69{,}12=70{,}63\Rightarrow T_2=139{,}75\ \text{J}$$

The angular velocity is obtained as $\omega=5{,}688$ rad

The velocity of the free end of the rod: $v_{A2}=\omega l=5{,}688\cdot 1{,}8=10{,}24\ \text{m/s}$

Example 5

A rod of mass $m=3$ kg and length $l=0.6$ m supported by a pin at one end is lift to a horizontal position then released.
Find the velocity of the centre of mass as well as the angular velocity when the rod reaches the position at $\varphi=20°$ as shown in the figure



Solution

Since the rod starts from rest, its initial kinetic energy is zero $(T_1=0\,\text{J})$.

Its angular velocity in the final position is ω_2, so kinetic energy is