8th PRACTICAL – CALCULATION OF MEAN DIRECTIONS AND ZENITH ANGLES FROM THEODOLITE MEASUREMENTS

After correctly setting up the theodolite and sighting a target point, we take readings on the horizontal and the vertical circle in both face left (FL) and face right (FR) positions.

1. Horizontal measurements

The following table shows part of the field book for the horizontal measurements with the measurement values filled in.

Note the column numbering in the second row. In the text, the columns will be referred to using the column numbers.

Horizontal measurements								
1	2	3	4	5	6	7	8	9
Station	Target	Readings					Mean direction	Collimation error
		FL°	‘	“	‘	“	Relative mean direction	
		FR°	‘	“	‘	“		
S	T	83	59	53				
				57				
		264	00	10				
				11				

Remember, that we have two second values in column 5, because after setting the coincidence with the micrometer screw on the theodolite and taking the first reading (55”), we adjusted the screw a little, set the coincidence again and took a second reading (57”).

Our first step is to calculate the mean of the minute and second values in columns 6 and 7. The arc minute part of the mean value goes into column 6, and the arc second part goes into column 7. (Remember, that we always round to the even number!)

Horizontal measurements								
1	2	3	4	5	6	7	8	9
Station	Target	Readings					Mean direction	Collimation error
		FL°	‘	“	‘	“	Relative mean direction	
		FR°	‘	“	‘	“		
S	T	83	59	53	59	55		
				57				
		264	00	10	00	10		
				11				

Next, we need to calculate the value of the collimation error in order to correct the measurements and calculate the mean direction. To compute the value of the error, we take the difference between the minute and second values of FL and the FR measurements. From columns 6 and 7, the values of the FL and FR measurements are the following:

$$\text{FL} = 59 - 55$$

$$\text{FR} = 00 - 10$$

The difference between them is 15” (if we add 5” to 59-55, we get 00-00 and then we have to add 10” more to get 00-10). So the value of the collimation error is 15” divided by 2, which is 7.5”. We have to

divide by 2 as we calculated the value using two measurements, but we only want to know the error of one measurement. When computing the errors, we do not round to integer arc seconds.

Now that we know the value of the error, we have to decide the sign of the error. We can decide this by taking a look at the minute and second values from columns 6 and 7. If we add the signed value of the collimation error to the FL measurement, we have get a value which is between the FL and the FR measurements. Using this line of thought, the sign of the collimation error has to be positive in this case, because if we add the 7.5" to 59-55, we get 00-02.5, which is between the FL and FR values.

Another way to calculate the collimation error with the correct sign is to use the following formula:

$$\delta_c = \frac{FR \pm 180° - FL}{2}$$

The $\pm$ after the FR value means, that we subtract 180° from the FR if FR > FL and add 180° if FR < FL.

We can fill this in the table:

Horizontal measurements								
1	2	3	4	5	6	7	8	9
Station	Target	Readings					Mean direction	Collimation error
		FL°	‘	“	‘	“	Relative mean direction	
		FR°	‘	“	‘	“		
S	T	83	59	53	59	55		+7.5”
				57				
		264	00	10	00	10		
				11				

The last step is to calculate the mean direction (MD). To do this, we take the FL value from columns 3, 6 and 7 and, add the value of the collimation error δ_c to it and round it to integer arc seconds:

$$MD = FL + \delta_c = (83 - 59 - 55) + (0 - 00 - 07.5) = 84 - 00 - 02.5 \approx 84 - 00 - 02$$

When we only have one direction, the relative mean direction is not applicable, we can leave the cell empty or fill in 0-00-00.

Horizontal measurements								
1	2	3	4	5	6	7	8	9
Station	Target	Readings					Mean direction	Collimation error
		FL°	‘	“	‘	“	Relative mean direction	
		FR°	‘	“	‘	“		
S	T	83	59	53	59	55	84-00-02	+7.5”
				57				
		264	00	10	00	10		
				11				

Below are some further examples. Now that we have measurements for multiple directions, we can calculate the relative mean direction. The relative mean directions denote the deflection angles between the first target and all the subsequent targets. The deflection angles are calculated by subtracting the mean direction of the first target from all the other targets. This means that the relative mean direction of the first target is zero, as its mean direction was subtracted from itself. The relative mean direction (RMD) of T2 will be:

$$RMD_{T2} = MD_{T2} - MD_{T1} = (8\text{-}36\text{-}39) - (321\text{-}12\text{-}21) = -312\text{-}35\text{-}42\ (+360°) = 47\text{-}24\text{-}18$$

Similarly, the relative mean direction of T3 is:

$$RMD_{T3} = MD_{T3} - MD_{T1} = (24\text{-}55\text{-}08) - (321\text{-}12\text{-}21) = -296\text{-}17\text{-}13 + (360°) = 63\text{-}42\text{-}47$$

Horizontal measurements								
Station	Target	Readings					Mean direction	Collimation error
		FL°	‘	“	‘	“	Relative mean direction	
		FR°	‘	“	‘	“		
S	T1	321	12	33 33	12	33	321-12-20	-12.5”
		141	12	05 10	12	08	0-00-00	
	T2	8	36	41 47	36	44	8-36-39	-5”
		188	36	32 35	36	34	47-24-19	
	T3	24	55	14 17	55	16	24-55-08	-8.5”
		204	54	59 59	54	59	63-42-48	

2. Vertical measurements

The table below contains the FL and FR reading from the vertical circle of the theodolite.

Vertical measurements								
1	2	3	4	5	6	7	8 9 10	11
Station	Target	Readings					FL+FR FL-FR z	Index error
		FL°	‘	“	‘	“		
		FR°	‘	“	‘	“		
S	T	88	42	21 22				
		271	17	35 30				

Similarly to the horizontal measurements, we first need to calculate the mean of minute and second values in columns 4 and 5, and write them in column 6 and 7.

Vertical measurements								
1	2	3	4	5	6	7	8 9 10	11
Station	Target	Readings					FL+FR FL-FR z	Index error
		FL°	‘	“	‘	“		
		FR°	‘	“	‘	“		
S	T	88	42	21	42	21		
				21				
		271	17	35	17	32		
				30				

The next step is to sum up the averaged FL and FR values and write it in cell no. 8. We also subtract the value of FR from FL and fill the result in cell no. 9. Keep in mind, that when working with readings from the vertical circle, the value of FL is always bigger than the value of FR, so if we subtract FR from FL, we get a negative angle, to which we have to add 360°.

Vertical measurements								
1	2	3	4	5	6	7	8 9 10	11
Station	Target	Readings					FL+FR FL-FR z	Index error
		FL°	‘	“	‘	“		
		FR°	‘	“	‘	“		
S	T	88	42	21	42	21	359-59-53	
				21			177-24-49	
		271	17	35	17	32		
				30				

We calculate the index error (δ_i) by taking the value of FL+FR in cell no. 8 and subtracting it from 360° and dividing it by 2:

$$\delta_i = \frac{360° - (359 - 59 - 53)}{2} = \frac{+7"}{2} = +3.5"$$

If the sum of the FL and FR values is bigger than 360°, then we get a negative index error as the result of the calculation above.

Vertical measurements								
1	2	3	4	5	6	7	8 9 10	11
Station	Target	Readings					FL+FR FL-FR z	Index error
		FL°	‘	“	‘	“		
		FR°	‘	“	‘	“		
S	T	88	42	21	42	21	359-59-53	+3.5”
				21			177-24-49	
		271	17	35	17	32		
				30				

Finally, we compute the zenith angle (z) by summing the value of the FL reading and the index error. We can check our calculations by dividing the FL − FR value in cell no. 9 by 2. If our calculations are correct, we have to get same zenith angle.

$$z = \mathrm{FL} + \delta_i = (88 - 42 - 21) + 3.5'' = 88 - 42 - 24.5 \approx 88 - 42 - 24$$

$$z = \frac{\mathrm{FL} - \mathrm{FR}}{2} = \frac{(177 - 24 - 49)}{2} = 88 - 42 - 24.5 \approx 88 - 42 - 24$$

Vertical measurements								
1	2	3	4	5	6	7	8 9 10	11
Station	Target	Readings					FL+FR FL-FR z	Index error
		FL°	‘	“	‘	“		
		FR°	‘	“	‘	“		
S	T	88	42	21	42	21	359-59-53	+3.5”
				21			177-24-49	
		271	17	35	17	32	88-42-24	
				30				

Further examples:

Vertical measurements								
Station	Target	Readings					FL+FR FL-FR z	Index error
		FL°	‘	“	‘	“		
		FR°	‘	“	‘	“		
S	T1	95	47	10	47	15	360-00-13	-6.5”
				20			191-34-17	
		264	12	58	12	58	95-47-08	
				59				
	T2	76	21	40	21	38	359-59-58	+1”
				37			152-43-18	
		283	38	21	38	20	76-21-39	
				20				
	T3	90	00	47	00	39	360-00-23	-11.5”
				31			180-00-55	
		269	59	41	59	44	90-00-28	
				46				