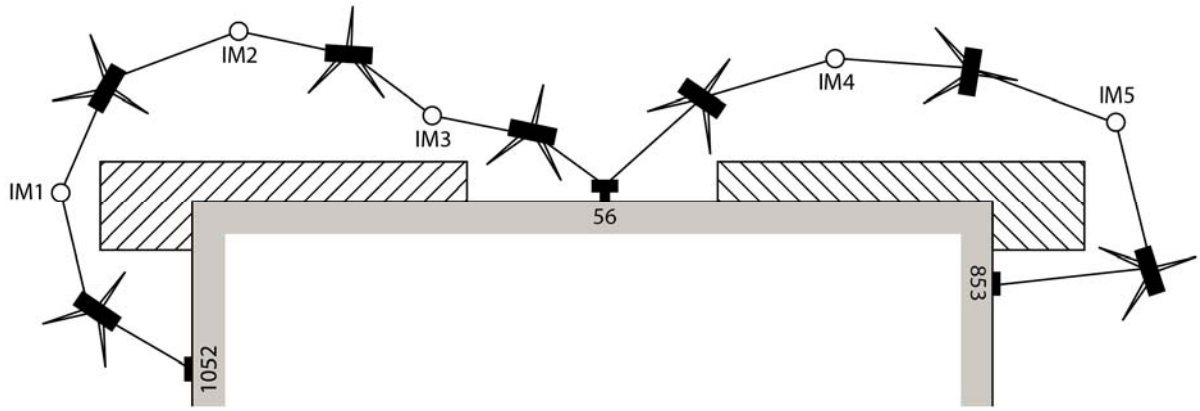
4th PRACTICAL – LINE LEVELING

The goal of this exercise is to determine the unknown elevation of point no. 56 by carrying out a line leveling between two points with known elevations (benchmarks, no. 1052 and no. 853) and including the point no. 56 in the line. A figure of the leveling line's layout:



The field books after taking the forward and backward measurements is as follows (only the numbers in black are measured, the numbers in blue are calculated values):

Forward direction					
Point ID	**Distance**	**Backsight**	**Foresight**	**Rise/Fall**	**Elevation [m]**
1052		1025			110.562
IM1	32	1432	0985	0040	
IM2	24	1785	1003	0429	
IM3	16	1126	1056	0729	
56	22	0988	0954	0172	
IM4	30	1550	1474	-0486	
IM5	40	1225	1635	-0085	
853	62		1047	0178	111.532

Backward direction					
Point ID	**Distance**	**Backsight**	**Foresight**	**Rise/Fall**	**Elevation [m]**
853		1423			110.562
IM6	64	1025	1253	0170	
IM7	42	1070	1109	-0084	
56	35	1368	1546	-0476	
IM8	20	2340	1199	0169	
IM9	16	1866	1630	0710	
IM10	19	1261	1398	0468	
1052	30		1238	0023	111.532

Please note that the elevation of the starting and the endpoint are already filled in, as these are known benchmarks. The Rise/Fall values between two points are calculated by taking the Backsight reading (BS) to the first point and subtracting from it the Foresight reading (FS) to the second point. For example, the rise value in the 3rd row of the first table is calculated by taking the BS to 1052 and subtracting the FS to IM1 (intermediate point no. 1). This gives us the elevation difference between the two points.

Also note that the distance in each row corresponds to the distance between the point denoted with the point ID in the same row and the point denoted with the point ID in the previous row.

As the leveling line was measured between two benchmarks, we can calculate the true elevation difference (Δh^{true}) between the two endpoints, using the elevations (H).

$$\Delta h^{true} = H_{end} - H_{start} = 111.538 - 110.562 = 970 \text{ mm}$$

By calculating the measured elevation difference and comparing it to the true elevation difference, we can calculate the misclosure of the line (Δ). For computing the measured elevation difference, we should create another table:

Point ID	Distance	Forward	Backward	Average	Correction	Corrected elevation difference	Elevation [m]
1052							110.562
	94	1370	-1372	1371	-2	1369	
56							**111.931**
	132	-0393	0397	-0395	-4	-0399	
853							111.532
Σ	226			0976	-6		

In the table above we filled in the summed distance between 620-56 and 56-852 from the "forward" field book because they are nearly equal to the summed distances from the "backward" field book. The next three columns contain the summed rise/fall values between the corresponding points (between 1052-56 and 56-953) and the average of the two values. Please note that when calculating the average of the forward and backward elevation differences, we have to change the sign of the backward elevation difference because of the direction difference.

By adding up the average elevation differences, we get that the measured elevation difference between the two endpoints (Δh^{meas}) is equal to 976 mm. To calculate the misclosure, we always take the true elevation difference and subtract from it the measured elevation difference:

$$\Delta = \Delta h^{true} - \Delta h^{meas} = 970 - 976 = -6 \text{ mm}$$

We somehow have to distribute the misclosure to the measured elevation differences. For this, we use the distances between the points as weights. We divide the misclosure by the sum of the distances and multiply with the corresponding distance.

$$\Delta_i = \frac{\Delta}{\sum d} \cdot d_i$$

The correction for the measured elevation diff. between 1052-56 and 56-853:

$$\Delta_{620-56} = -\frac{6}{226} \cdot 94 = -2.495 \approx -2 \text{ mm}$$

$$\Delta_{56-853} = -\frac{6}{226} \cdot 132 = -3.504 \approx -4 \text{ mm}$$

We compute the elevation of point no. 56 by taking the elevation of the starting point (1052) and adding the corrected elevation difference between 1052-56 ($\Delta h^c_{1052-56}$). Alternatively, we can take the elevation of the endpoint and subtract from it the elevation difference between 56-853 (Δh^c_{56-853}). We need subtraction in these case as we are going "backwards" from the endpoint, whereas in the first we are going "forward" from the starting point.

$$H_{56} = H_{1052} + \Delta h^c_{1052-56} = 110.562 + 1.369 = 111.931 \text{ m}$$

$$H_{56} = H_{853} - \Delta h^c_{56-853} = 111.532 - (-0.399) = 111.931 \text{ m}$$

Please note that the elevation differences in the table are in mm unit, while the elevations are in m unit. This means that we have to convert the differences into meters when we add them to the elevations.